DO NOW

Have you ever heard of optimum size, greatest profit, greatest distance, or greatest strength?

What do these ideas center around or all have in common?

Can you picture a connection between a recent calculus procedure and finding these points (or things)?

4.7 Optimization Problems

Common application of calculus to determine the minimum and maximum values.

For Example: greatest profit
least cost
least time
greatest voltage
optimum size
least size
greatest strength
greatest distance

Procedure:

1. Identify all ***given*** quantities and quantities ***to be determined***. If possible, make a sketch.
2. Write a **primary equation** for the quantity that is to be maximized or minimized.
3. Reduce the primary equation to one having a ***single independent variable***. This may involve the use of **secondary equations** relating the independent variables of the primary equation.
4. Determine the feasible domain of the primary equation.
5. Find the derivative of the primary equation and find its critical numbers.
6. Use the first and/or second derivative tests to determine the maximum or minimum. CLEARLY identify the appropriate answer(s).

Example:

1. Find two positive numbers whose product is 192 and whose sum is a minimim.

primary equation: $S=x+y$

Secondary equation: $xy=192$

$y=\frac{192}{x}$

$S=x+\frac{192}{x}$ domain: $x>0$

$S'=1-\frac{192}{x^2}$

$S'=\frac{x^2-192}{x^2}$

$x^2-192=0$ or $\cancel{x^2=0}$ not in domain

$x=\pm\sqrt{192}=\pm 8\sqrt{3}$

$-\sqrt{192}$

$(0, 8\sqrt{3}) \mid (8\sqrt{3}, \infty)$

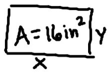
$f'(1)=- \mid f'(14)=+$

minimum

$x=8\sqrt{3}$ $y=8\sqrt{3}$

$\boxed{8\sqrt{3}, 8\sqrt{3}}$

2. What is the smallest perimeter possible for a rectangle whose area is 16in^2, and what are its dimensions?



$P=2x+2y$ ←primary

$xy=16$ ←Secondary

$y=\frac{16}{x}$

domain: $x>0$

$P=2x+2\left(\frac{16}{x}\right)$

$P=2x+\frac{32}{x}$

$P'=2-\frac{32}{x^2}$

$P'=\frac{2x^2-32}{x^2}$

$2x^2-32=0$ $\cancel{x^2=0}$ Not in domain

$x=\pm 4$

-4 not in domain

$(0,4) \mid (4,\infty)$

$f'(1)=- \mid f'(5)=+$

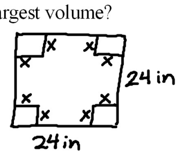
minimum

$x=4$ $y=4$

$P=2(4)+2(4)$

$\boxed{P=16 \text{ in} \quad 4 \text{ in} \times 4 \text{ in}}$

3. A box manufacturer wishes to make open boxes from material that is 24 in square by cutting equal squares from the four corners and turning up the sides. What size square should be cut from each corner to obtain the box with the largest volume?



domain: $0<x<12$

$V=lwh$

$V=(24-2x)(24-2x)x$

$V=(576-96x+4x^2)x$

$V=4x^3-96x^2+576x$

$V'=12x^2-192x+576$

$V'=12(x^2-16x+48)$

$x^2-16x+48=0$

$(x-12)(x-4)=0$

$\cancel{x=12}$ $x=4$

Not in domain

$V''=24x-192$

$V''(4)=24(4)-192$

$V''(4)=-$ ⌢ max.

$x=4$

$\boxed{4 \text{ in} \times 4 \text{ in}}$

HOMEWORK

Worksheet - HW 4.7.1